Large-Scale Motion in the Venus Stratosphere[1]

WILLIAM B. ROSSOW[2]

Geophysical Fluid Dynamics Program, Princeton University, Princeton, NJ 08540

GARETH P. WILLIAMS

Geophysical Fluid Dynamics Laboratory/NOAA, Princeton University, Princeton, NJ 08540

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ABSTRACT

To examine a postulate that the horizontal momentum exchanges in Venus' stratosphere are quasi-nondivergent, we investigate the properties of two-dimensional turbulence on a slowly rotating sphere in a high-resolution, one-level numerical model. We conclude that the forcing which maintains the stratospheric flow is weak and influences the dynamics far less than the inertial effects. Consequently, the behavior resembles that of vorticity-conserving, two-dimensional flows. On a slowly rotating planet like Venus, such flows are dominated by solid-body rotation and by a planetary wave of unit zonal wavenumber—this wave corresponds to the observed Y-shaped UV feature. Although these largest scales of motion stand out, the dynamic balances of the flow are fundamentally nonlinear, in contrast to the quasi-linear Rossby wave regime on rapidly rotating planets.

1. Introduction

We are concerned with explaining the horizontal distribution of angular momentum in the Venus stratosphere. Two properties of slowly rotating atmospheres suggest that the momentum exchanges are quasi-nondivergent and consequently two-dimensional: 1) in the absence of a geostrophic constraint on the dynamics, the hydrostatic balance and a highly stable stratification force motions to follow constant density surfaces; 2) when differential heating drives the atmosphere, the circulation tends to be of a quasi-barotropic Hadley form, in which the constant density surfaces lie almost horizontally. To understand the consequences of such constraints for Venus, we investigate the properties of two-dimensional turbulence on a slowly rotating sphere. The study complements previous analyses of two-dimensional turbulence in different stationary and rotating domains (Lilly, 1972a,b; Rhines, 1975, 1977; Williams, 1975, 1978) and provides a useful prelude to a more complete simulation—an act hindered by the current inadequacy of data.

Observations of the upper atmosphere of Venus have revealed an unexpected distribution of angular momentum, in which planetary-scale waves are superimposed on a predominantly zonal flow of nearly uniform angular velocity; this relative angular rotation exceeds the planetary rotation by two orders of magnitude. On other scales, power spectra analyses of the sizes of cloud features, when interpreted as kinetic energy spectra, produce characteristics symptomatic of two-dimensional turbulence (Travis, 1978). These results emerge from a variety of observations and techniques: ground-based UV photographs (Dollfus, 1975), spectroscopic Doppler measurements (Traub and Carleton, 1975, 1978), Venera series entry probes (Marov *et al.*, 1973; Kerzhanovich and Marov, 1977) and detailed tracking of small-scale features in the Mariner 10 UV photographs (Suomi, 1974; Belton *et al.* 1976a; Limaye and Suomi, 1977). Unfortunately, our knowledge of the angular momentum distribution is incomplete because of observational limitations, e.g., the Mariner 10 photographs are confined to low latitudes (mostly in the Southern Hemisphere on the sunlit side), and the longitudinal flow structure can only be inferred from the data by assuming the time dependence of the flow (cf. Belton *et al.*, 1976a; Limaye and Suomi, 1977). Such ambivalence precludes detailed comparison with model results.

Previous attempts to understand what processes maintain this remarkable angular momentum distribution have concentrated on simple models of the *vertical* transport of the planet's angular momentum into the upper atmosphere. The model of Fels and Lindzen (1974) (also, Fels, 1977) involves vertical

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[2] Presently at Goddard Institute for Space Studies, New York, NY 10023.

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transport by vertically propagating internal gravity waves excited by the zonal variation of solar heating, while the model of Gierasch (1975) involves vertical transport by an axisymmetric Hadley circulation driven by the meridional variation of solar heating. The latter of these two models also requires some process of horizontal momentum transport which Gierasch represents by an extremely large horizontal eddy viscosity; however, Kálnay de Rivas (1975) has shown that if the Prandtl number of the eddies is unity, such a strong mixing also eliminates the temperature gradient that drives the Hadley circulation. What is needed is some dynamical process which transports momentum, but not heat, horizontally. The similarity between the horizontal distribution of angular momentum obtained by Rossby (1947) and that observed in Venus' stratosphere led Gierasch (1975) to suggest Rossby's "vorticity mixing" as an alternative to the momentum mixing.

In his attempt to understand the horizontal angular momentum distribution in Earth's atmosphere, Rossby (1947) introduced the concept of vorticity mixing in a one-level model atmosphere. Rossby's model was not successful because it conserved absolute vorticity and, therefore, it did not account for the baroclinic and surface processes which create and destroy vorticity. However, subsequent research has shown that Rossby's model equation, the vorticity equation, with the addition of forcing terms to represent the most important baroclinic and surface processes, simulates rather well the dynamics of Earth's atmosphere and ocean, as well as Jupiter's upper atmosphere [see Williams (1978) for a fuller discussion]. Such one-level models apply to rapidly rotating atmospheres because, in the latter, the geostrophic constraint produces quasi-nondivergent flow along quasi-horizontal isobars so that the real and model systems have analogous two-dimensional motions—even though their density structures differ. However, the slow rotation of Venus imposes no geostrophic constraint on the dynamics so the analogy between geostrophic and two-dimensional motions does not apply; for nondivergent flow to exist, the stratosphere has to be essentially barotropic. A further consequence of this near barotropy is that the two-dimensional motions do not transport heat efficiently.

Scale analysis of the equations for thermally forced motions under Venus conditions (Section 2) supports the idea that the stratospheric motions are nearly divergence-free and two-dimensional. A similar result holds for the large-scale circulation in Earth's tropics (Charney, 1963; Holton, 1972) although planetary-scale, vertically propagating gravity waves can transport significant momentum and energy (Holton, 1969). Fels and Lindzen's model of the 4-day wind involves such wave action. We represent the effects of gravity waves or of weak thermal circulations by a simple forcing of a two-dimensional model. The flows resulting from weak forcing closely resemble those observed, with the almost-uniform angular momentum distribution and the Y-shaped UV feature being natural consequences of two-dimensional dynamical processes.

The numerical procedure used to solve the model equations is presented in Section 3. This is followed by a discussion of the nature of vorticity conserving dynamics (Section 4) and by examples of the flow patterns produced when vorticity sources and sinks are present (Section 5). Finally, we discuss the implications of our results (Section 6).

2. Scale analysis of the vorticity equation

The general Navier–Stokes equations of fluid motion in a rotating reference frame reduce to the familiar vorticity, divergence and hydrostatic equations when the so-called traditional approximations are made (Lorenz, 1967). In particular, the prognostic equation for the vertical velocity is replaced by the hydrostatic relation, with the vertical velocity determined diagnostically from the continuity equation. The horizontal momentum equations are equivalent to the vorticity and divergence equations. The vorticity equation in spherical, log-pressure coordinates is

$$\zeta_t+\mathbf{U}\cdot\nabla_2(\zeta+f)+\omega\zeta_z = -(\zeta+f)\delta+\nabla_2\omega\cdot(\mathbf{k}\times\mathbf{U})_z+\nu\nabla_3^2\zeta, \quad (1)$$

with the following notation:[3]

(λ,θ)	longitude, latitude
(u,v)	corresponding velocity components of $\mathbf{U}$, the horizontal wind velocity
z	vertical coordinate $[=\ln(p_s/p)]$, $\mathbf{k}$=unit vertical coordinate vector
p, p_s	pressure, surface pressure
ω	vertical velocity $[=Dz/Dt]$
ζ	vertical component of relative vorticity $\{=(1/a\cos\theta)[v_\lambda-(u\cos\theta)_\theta]\}$
f	vertical component of planetary vorticity $[=2\Omega\sin\theta]$
δ	horizontal divergence $\{=(1/a\cos\theta)[u_\lambda+(v\cos\theta)_\theta]=-e^z(e^{-z}\omega)_z\}$
ν	viscosity parameter
∇_2	del operator on constant z surfaces
∇_3	three-dimensional del operator.

The first two terms on the right in Eq. (1) represent, respectively, the creation/destruction of vorticity through the contraction/expansion of vortex tubes by the divergence of the horizontal flow and the twisting/stretching of vortex tubes by the shears in the flow. The third term represents vorticity diffusion by unresolved scales of motion. One important consequence of this equation is that for all rotating differentially heated atmospheres, $f\delta$ is nonzero, and so these atmospheres are always vorticity full.

[3] Subscripts t, z, λ and θ indicate differentiation with respect to these coordinates.

For Venus, the insignificance of the rotational terms in (1) give its stratosphere a unique dynamics. We scale the equation according to the following scheme:

$$\left.\begin{aligned}(a\cos\theta\partial\lambda, a\partial\theta) &= a(\cos\theta\partial\lambda,\partial\theta), \quad & t &= (a/U)t^*\\ (u,v) &= U(u^*,v^*), & z &= Hh\\ \zeta &= (U/a)\zeta^*, & \omega &= W\omega^*\\ & & \delta &= (W/H)\delta^*\end{aligned}\right\}, \quad (2)$$

where $H\sim 1$ is the depth of the cloud-filled stratosphere in scale heights. Substituting (2) into (1), multiplying by $(a/U)^2$, and dropping the asterisks, yields

$$\zeta_t+\mathbf{U}\cdot\nabla_2\zeta = -(Wa/U)[\omega\zeta_h+\zeta\delta-\nabla_2\omega\cdot(\mathbf{k}\times\mathbf{U})_h] + (\nu/Ua)\nabla_3^2\zeta. \quad (3)$$

In the Venus stratosphere, the near balance between adiabatic cooling and radiative heating (Gierasch, 1975) suggests that $W\approx 10^{-7}$ s^{-1} so that $Wa/U\ll 1$. (This inequality may not be so strong if motions driven by diurnal temperature contrasts are important.) The vorticity source/sink contributions of thermally forced motions in the stratosphere are, therefore, not of leading order. The balancing of these source terms by dissipation implies that the diffusion term is also secondary, and mixing length theory supports this conclusion. Therefore, to leading order, the dynamics of Venus' stratosphere is consistent with the conservation of relative vorticity, namely,

$$\zeta_t+\mathbf{U}\cdot\nabla_2\zeta = 0, \quad (4)$$

i.e., *the forcing which maintains the dynamics of the stratosphere is weak in magnitude and only weakly coupled to the flow.*

The above estimate of W is only valid for the thermally forced motions. We have no way of estimating the value associated with any mechanically or thermally excited inertial oscillations that could produce a strong forcing and a strong coupling between different levels of the atmosphere (cf. Fels, 1977). In this investigation, we have chosen to study the response of the stratosphere modelled as a two-dimensional fluid layer that obeys (4) (in the absence of forcing and dissipation) or (3) with the first term on the right represented as a simple forcing function, dependent on time and location only.

3. The numerical model

a. Equations

We consider a two-dimensional fluid and introduce the standard velocity streamfunction ψ. Eq. (3) becomes

$$\zeta_t+\psi_x(\zeta+f)_y-\psi_y\zeta_x = \nu\nabla_2^2\zeta+F-D, \quad (5)$$

with the following notation:

(x,y)	prograde longitudinal and poleward latitudinal coordinates $[=(a\lambda, a\sin\theta)]$
m	mapping factor $[=\sec\theta]$
(u,v)	$(-m^{-1}\psi_y, m\psi_x)$
f	$2\Omega y/a$
ζ	$mv_x-(u/m)_y=m^2\psi_{xx}+(m^{-2}\psi_y)_y=\nabla_2^2\psi$
∇_2^2	Laplacian operator
F	forcing term
D	drag term $[=(\zeta\sin^4\theta)/\tau_D]$
τ_D	time scale of drag.

Eq. (5) is solved on a (128×128) grid-point array using the Arakawa (1966) finite-difference scheme which has the energy and enstrophy conserving properties essential for accurate and stable simulation of turbulent flows [see Williams (1978) for more details of the numerical scheme]. The time stepping is performed by a standard leapfrog method.

b. Dissipation

There are two types of dissipation in these calculations. The drag D is formulated as an Ekman-like surface drag, but only represents a convenient way to limit the amplitude of the vorticity and control the integration time required for equilibration of the flow. This formulation of the drag caused by the surface and lower atmosphere is not realistic; but since this term is not scale selective and only weakly dependent on latitude, it does not significantly alter the qualitative pattern of the solutions obtained.

A simple Laplacian diffusion represents the small-scale horizontal mixing process. (The more complicated operator needed to conserve angular momentum in solid-body rotation is not necessary when the diffusion time scale is much longer than the integration time, as is the case here.) The coefficient ν has the smallest value consistent with proper spectral representation and minimal dissipation (see Williams, 1978).

c. Boundary conditions

The integrations are made on global sectors with periodic boundaries at longitudes $\lambda=0$, λ_0 and impermeable boundaries $\psi_x=\psi'=0$ at latitudes $\theta=\pm\theta_0$. Usually, $\lambda_0=360°$ and $\theta_0=70°$. ψ' is the variation of ψ from its zonal mean value $\bar{\psi}$.

The boundary conditions on the impermeable walls maintain a condition of "wall neutrality" (boundary conditions BC2 in Williams, 1978) and conserve the vorticity and angular momentum of the flow within the accuracy of the sidewall truncation errors.

d. Formulation of F

We consider three types of forcing functions: axisymmetric, localized and pseudo-baroclinic. The axisymmetric forcing functions were chosen to produce

zonal flows with various meridional structures, usually symmetric about the equator; these functions mimic the behavior of such mechanisms as a Hadley circulation in supplying momentum to the stratosphere (e.g., Gierasch's model). The localized forcing functions produce vorticity in a latitudinally and longitudinally limited area centered about the equator that moves with the sun's surface velocity and mimics a thermally excited momentum source (e.g., Fels' model); however, this motion does not appear to play an important role in the equilibrated flow. The pseudo-baroclinic forcing represents the effect of baroclinic eddies by introducing *kinetic energy* into barotropic eddies of a specific wavelength and random phase (see Williams, 1978 for details)—in contrast to the other forcing functions which inject *momentum and kinetic energy* into jets. Some of the 26 different cases studied appear below.

4. Vorticity conservation and two-dimensional turbulence

We gain a better understanding of the dynamics of two-dimensional flows with vorticity sources and sinks (Section 5) by first studying motions with conserved absolute vorticity

$$\zeta_t+\psi_x(\zeta+f)_y-\psi_y\zeta_x=0. \tag{6}$$

A very large number of studies of Earth's atmospheric and oceanic dynamics take (6) as an approximation of the full equations of motion or use it to isolate the so-called barotropic processes. We briefly discuss some of these studies to illustrate the context of our numerical solutions of (6).

Rossby's (1947) solution for a steady-state atmosphere is not a solution of the homogeneous equation (6), since he assumes a constant equatorward flux of vorticity below some latitude. However, in his preliminary discussion, he states "that for a prescribed momentum, a minimum of kinetic energy is reached when the entire (atmosphere) rotates as a solid"—in the absence of vorticity sources and sinks—and that this atmosphere "cannot by itself give rise to lateral mixing processes, since no energy is available for that purpose."[4] This suggests 1) that solid rotation is the only "stable" state of a two-dimensional vorticity-conserving flow in the sense that this state cannot evolve further under these constraints; 2) that an atmosphere initially having some other angular velocity distribution would "relax" to solid rotation in the absence of any vorticity sources or sinks. However, since other angular velocity distributions contain more kinetic energy than does the solid rotation state, their "relaxed" or "stable" state must contain waves.

To define the waves on Venus, it is useful to consider Thompson's (1948) extension of the classical Rossby–Haurwitz wave theory (Rossby, 1939; Haurwitz, 1940). This shows that for latitudinally varying mean flows, a special class of waves of *permanent shape* can exist, with a dispersive phase speed given by

$$c=-\bar{\psi}_y-\frac{f_y-\bar{\psi}_{yyy}}{k^2+m^2}, \tag{7}$$

where k and m are the zonal and meridional wavenumbers, $\bar{\psi}_y$ the basic zonal velocity and the coordinates Cartesian. When the planetary rotation is negligible, as on Venus, dispersive wave solutions to (6) can still occur because, as (7) indicates, the meridional gradient in the vorticity of the mean flow acts like the planetary vorticity gradient. Such waves are linearized advection waves, i.e., waves that advect relative vorticity rather than planetary vorticity, in contrast to the Rossby waves. In the special case where the mean zonal flow resembles solid rotation, (7) can be reduced by a coordinate transformation to the Rossby wave solution; but in general this analogy does not exist.

The numerous linear stability analyses of different forms of mean zonal flow or waves suggest which solutions of (6) form the stable or relaxed state. For two-dimensional flows, the instability has been called the barotropic or shear instability. One class of such stability problems concerns the transfer of energy from a mean zonal flow into small, growing, wavelike disturbances. The mean zonal flow on a sphere must meet two conditions for instability (Fjortoft, 1950; Kuo, 1951): 1) conservation of kinetic energy requires that the integral over the sphere of the product of velocity and vorticity gradient be positive, and 2) conservation of angular momentum requires that the absolute vorticity change sign somewhere on the sphere. The first condition requires that the disturbance acquire its kinetic energy from the mean flow by redistributing the angular momentum of the mean flow. This leads naturally to Rossby's statement that the solid rotation state must be stable against shear instability. However, because of the amplitude dependence in the second condition, small deviations from large-amplitude solid rotation can also be stable. This is the well-known "β-effect" on a rapidly rotating planet which allows strong jetlike structure in the zonal flow relative to the surface because it is superimposed on a much larger rotational zonal flow. When viewed from an inertial reference frame, the deviation from solid rotation caused by these "jets" is normally small. However, on Venus, with its small rotation rate, any strong jetlike structures in the zonal flow relative to the surface must be barotropically unstable and, consequently, maintained by some process.

The other class of stability problems concerns the transfer of energy from a Rossby–Haurwitz wave into small, growing disturbances which can include zonal flows or other Rossby–Haurwitz waves. Such

[4] This minimum energy statement can easily be proved using the method of Lagrange multipliers (P. J. Gierasch, private communication).

Rossby–Haurwitz wave interactions at small amplitudes are resonant interactions of second order; i.e., they must involve three wave components which satisfy the resonance condition (Hoskins, 1973, Baines, 1976). *The conditions for the barotropic instability of Rossby–Haurwitz waves parallel those for zonal flows* (Baines, 1976): (i) *an energy condition defines a set of waves, those with wavenumbers less than three, which are stable regardless of amplitude, and (ii) an amplitude condition produces a "β-effect" on rapidly rotating planets that stabilizes waves with intermediate wavenumbers, while all smaller scales are unstable.* Although we must consider the more general advection waves on Venus, these results suggest that only the largest waves are barotropically stable.

Turbulence studies, in contrast to the stability analyses, consider the evolution of a complete spectrum of motion, containing all zonal and wave components without focusing on particular interactions. The conclusions from studies of two-dimensional turbulence are qualitatively similar to and support those of the stability analyses. The integral constraints on (6), namely, conservation of kinetic energy, angular momentum, and enstrophy, together with the plausible requirement that the nonlinear interactions broaden an initially sharp kinetic energy spectrum, require that the majority of the kinetic energy be transferred to the larger scales, while the majority of the enstrophy is transferred to the smaller scales (Fjortoft, 1953; Merilees and Warn, 1975; Baines, 1976). For a particular wave component or scale of motion, the nonlinear interaction is second order and must involve one larger and one smaller wavelength component. Consequently, on a finite domain, such as a sphere, where the wave spectrum is discrete, the very largest scale waves must be stable since they cannot give up energy to smaller scale waves by these second-order interactions. Smaller scale waves are "unstable" in that they give up most of their energy to larger scale waves. The "relaxed" state that results is still turbulent, containing all the scales of motion, but the lowest wavenumber zonal flow and advection wave now completely dominate the motion.[5]

All of these suggestions come together in recent numerical simulations of two-dimensional, vorticity conserving motions. The more complicated case of rapidly rotating fluids has been considered as a model of the dynamics of Earth's oceans by Rhines (1975, 1977) and of Earth's and Jupiter's atmospheres by Williams (1975, 1978). Briefly, they find that the character of the wave and zonal flow interactions changes when the scale of the motions becomes large enough that the Coriolis gradient β becomes effective. At smaller scales, the full nonlinear interactions allow energy-enstrophy exchanges among all components with the consequent rapid cascade of kinetic energy to larger scales and enstrophy to smaller scales where even extremely weak viscous effects can dissipate it. However, at the larger scales, on entering the Rossby wave regime, the more restrictive constraints of resonant interactions strongly inhibit the energy cascade. Energy advection is replaced by energy radiation by the propagating Rossby–Haurwitz waves (Rhines, 1977). The "relaxed" state of a fully turbulent, two dimensional flow on a rapidly rotating planet contains a complex of propagating "stable" Rossby–

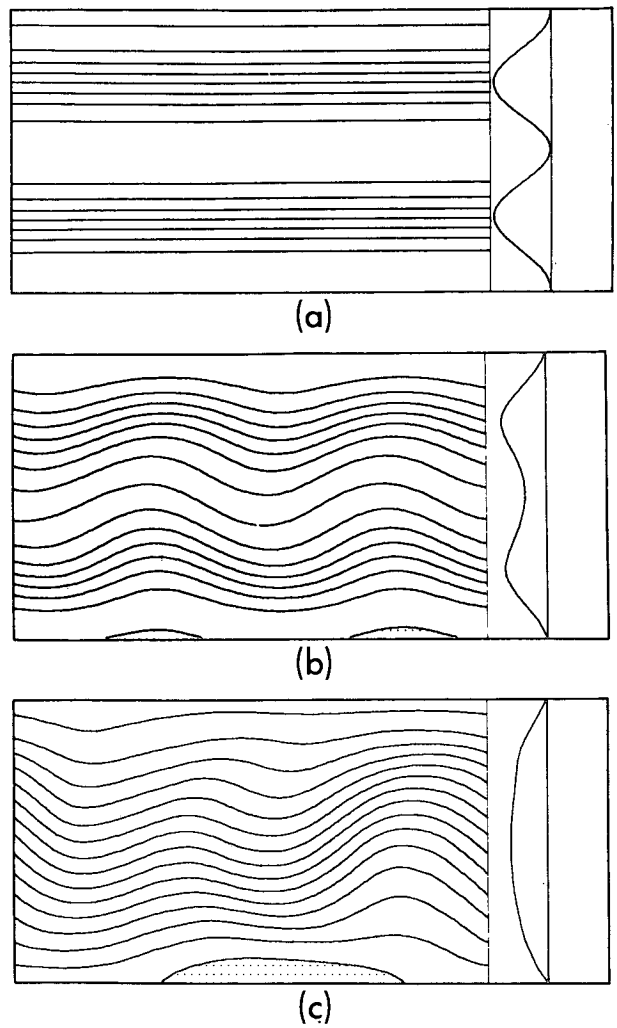

FIG. 1. Contours of the flow streamfunction ψ in x, y coordinates (see text) and the meridional profile of the mean zonal flow velocity $\bar{u}$. Negative ψ is indicated by shading and the contour interval is $\Delta\psi$. The scale of $\bar{u}$ is u^*. The accumulated integration time, in days, is t. (a) Case A1, $\Delta\psi = 40$ km^2 s^{-1}, $u^* = 0.1$ km s^{-1}, $t = 2.3$ days; (b) case A2, $\Delta\psi = 40$ km^2 s^{-1}, $u^* = 0.1$ km s^{-1}, $t = 13.8$ days; (c) case A1 as (a), $t = 13.8$ days.

[5] Baines (1976) finds two stable zonal flows and two stable planetary waves; however, those modes which give velocity fields with odd symmetry about the equator only predominate in the "relaxed" state of a flow with no net angular momentum. Since we consider only flows with net angular momentum, here, the even symmetry zonal flow and planetary wave modes predominate.

TABLE 1. Parameter values for numerical models discussed. Cases labeled A have no forcing and represent the free inertial development of some initial flow state. Cases labeled B have a homogeneous pseudobaroclinic forcing. Cases labeled C have a forcing function localized in both longitude and latitude. Cases labeled D have axisymmetric forcing. The dissipation time scale $\tau_\nu = a^2/\nu$, where a is the planetary radius ($=6000$ km). Venus' rotation rate $= -0.3\times10^{-6}$ s^{-1}.

Model	Figure no.	Integration time (days)	τ_D(day)	τ_ν(days)
A1[a]	1	19.5	∞	1830
A2[a,b]	1	16.7	∞	1830
B1	2	78.2	20.0	3660
C1	3	46.0	5.0	3660
C2	4	54.6	5.0	1220
C3	7	51.7	20.0	3660
D1[a]	5	23.0	5.0	3660
D2[a,c]	6	24.1	5.0	1830

[a] $\lambda_0 = 180°$.
[b] The planetary rotation rate is that of Earth, with sign reversal.
[c] $\theta_0 = 60°$.

Haurwitz waves and a mean zonal flow with jetlike structure (Williams, 1978).

The free inertial development of two-dimensional turbulence for a non-rotating fluid has been simulated by Lilly (1972a,b). We have investigated several examples of this evolution for fluid on a slowly rotating (Venus-like) sphere. The first example (Fig. 1) illustrates the barotropic instability of axisymmetric jets and the role of the β effect. In these calculations, a forcing function F is introduced in (5) with $D=0$ in order to produce the desired initial flow (Fig. 1a). After this "spin-up," F is set to zero at 2.3 days and the calculation proceeds with only weak dissipation (see Table 1). For axisymmetric flows, a small (1% of the total kinetic energy) white noise perturbation is introduced in the next time step to initiate instability. Under the large terrestrial rotation rate (reversed in sign) (Fig. 1b), the initial jet (with a velocity much larger than typical on Earth) is weakly unstable and produces some small amplitude waves with a zonal wavenumber ~2 and a small increase in the eddy kinetic energy; but even after 11 days, the jet structure remains intact. Under the slow Venus' rotation rate, Fig. 1c, the initial perturbations grow rapidly and completely obliterate the jet structure in ~6 days. After 14 days, the flow is one of solid rotation with very weak, large-scale (wavenumber ~1 over the domain) planetary waves.

We have obtained comparable results for several other zonal flows with Venus' rotation rate. In particular, we found the zonal flow $u=u_0\cos^2\theta$ breaks down in only a few days, while the flow $u=u_0\cos\theta$, solid rotation, is stable for 43 days, even when repeatedly perturbed by a white noise spectrum containing 10% of the total kinetic energy. This confirms, though not rigorously, the conclusions of Rossby, and the theories of linear barotropic instability and two-dimensional turbulence.

One consequence of these results for numerical modeling is that if the calculation domain is smaller than the wavelength of the most unstable mode, the flow can be artificially stabilized. This is illustrated by our results for $u=u_0\cos^2\theta$ for which the most unstable mode has a zonal wavenumber of approximately 2 (Baines, 1976). We found that this flow was stable for all calculations on a global domain covering less than 360° of longitude. Severe truncation of the motion spectrum by coarse horizontal resolution can produce a similar effect (Puri and Bourke, 1974, Baines, 1976).

The second example (Fig. 2) illustrates the barotropic instability of large wavenumber flows. In this case, a pseudo-baroclinic forcing (Williams, 1978) produces eddies with latitudinal and longitudinal wavenumbers of 5 and 7, respectively (Fig. 2a). Linear theory predicts that these waves should be unstable. Indeed, after ~32 days the flow has evolved by two-dimensional processes from these simple eddies into a flow containing all scales of motion but with the majority of the kinetic energy in the lower wavenumbers (Fig. 2b)—even though the forcing at wavenumber 7 remains evident (Fig. 2e). At 32 days F and D are set to zero and the flow evolves freely with the result shown in Figs. 2c and 2f.[6]

All the calculations lead to the conclusion that two-dimensional vorticity conserving exchanges on a slowly rotating planet like Venus produce a "relaxed" state consisting of a solid rotation plus the largest planetary-scale advection wave (when the angular momentum is nonzero). We can speak of a "stable" or "relaxed" flow because the cascade of enstrophy to smaller scales leads to dissipation, even by small ν values, making the evolution irreversible. Furthermore, stability analyses (Baines, 1976) define which zonal flows and planetary waves comprise the "stable" state so that the *stable state is unique.* The "relaxed" state of a non-rotating planet is, at once, "smoother" but more nonlinear than the "relaxed" state attained on a rapidly rotating planet (cf. Williams, 1978).

The "relaxation" time τ_0 determined by the nonlinear processes lacks a simple definition, but for Venus' kinetic energy level and spectral distribution it is ~15 days. Since the dissipation time scale τ_ν (see Table 1) greatly exceeds τ_0, even weak forcing can "pump" the flow to large energy levels. This last property of two-dimensional turbulence is both the key to and difficulty in understanding forced flows—as we shall see in Section 5.

5. The forced vorticity equation: Numerical results

We now consider some examples of solutions to (5) with the continuous forcing, drag and viscosity active

[6] See the previous footnote.

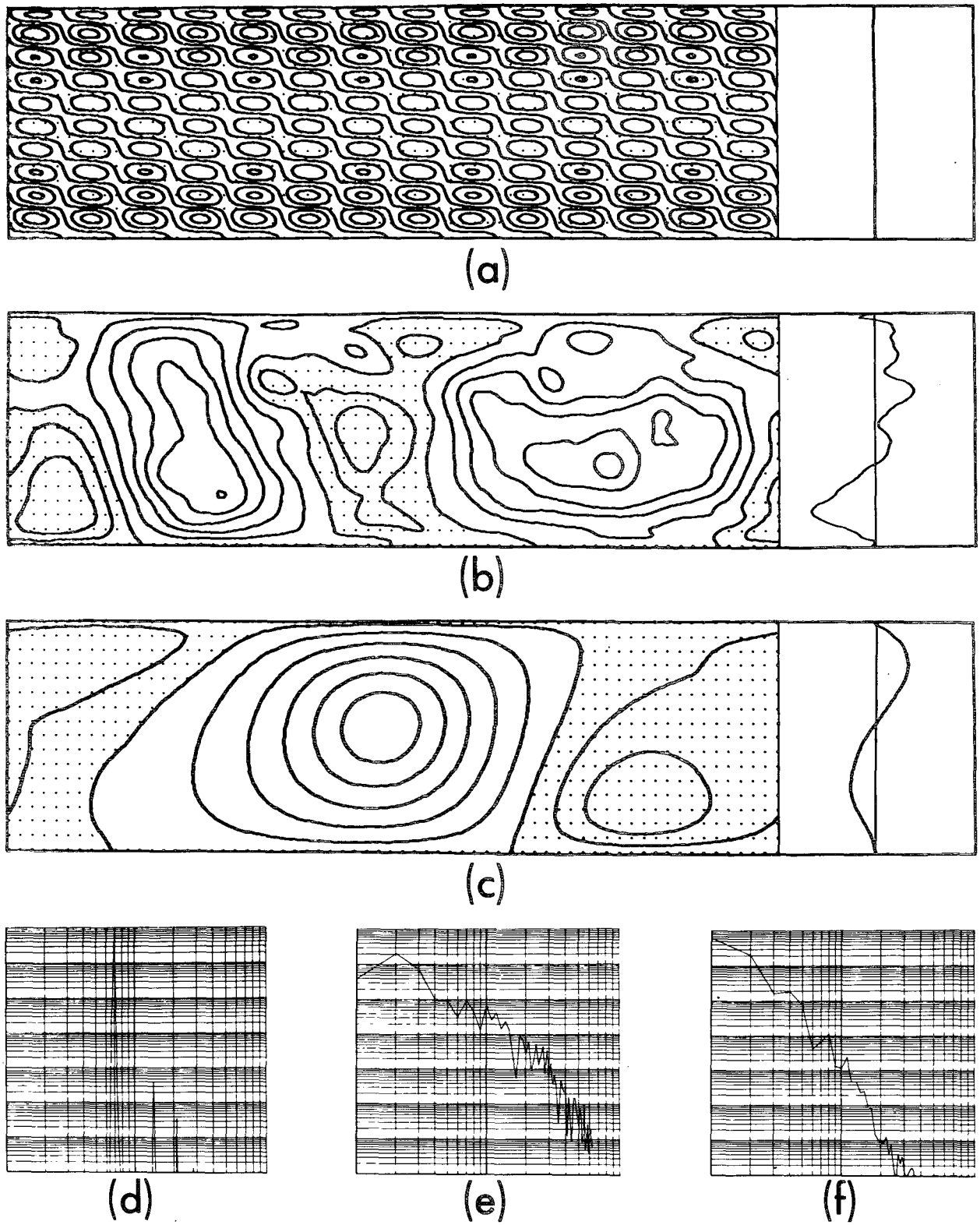

FIG. 2. Case B1: (a) Contours of ψ and profile of $\bar{u}$. $\Delta\psi = 2$ km^2 s^{-1}, $u^* = 0.05$ km s^{-1}, $t = 1.3$ days; (b) as in Fig. 2a except $\Delta\psi = 50$ km^2 s^{-1}, $t = 32.2$ days; (c) as in Fig. 2b except $t = 78.2$ days; (d) Normalized zonal spectrum of the kinetic energy on latitude circle at $\theta = -28°$. The ordinate is in arbitrary units and the abscissa is the zonal wavenumber from 1 to 100. $t = 1.7$ days; (e) as in Fig. 2d except $t = 32.2$ days; (f) as in Fig. 2d except $t = 78.2$ days.

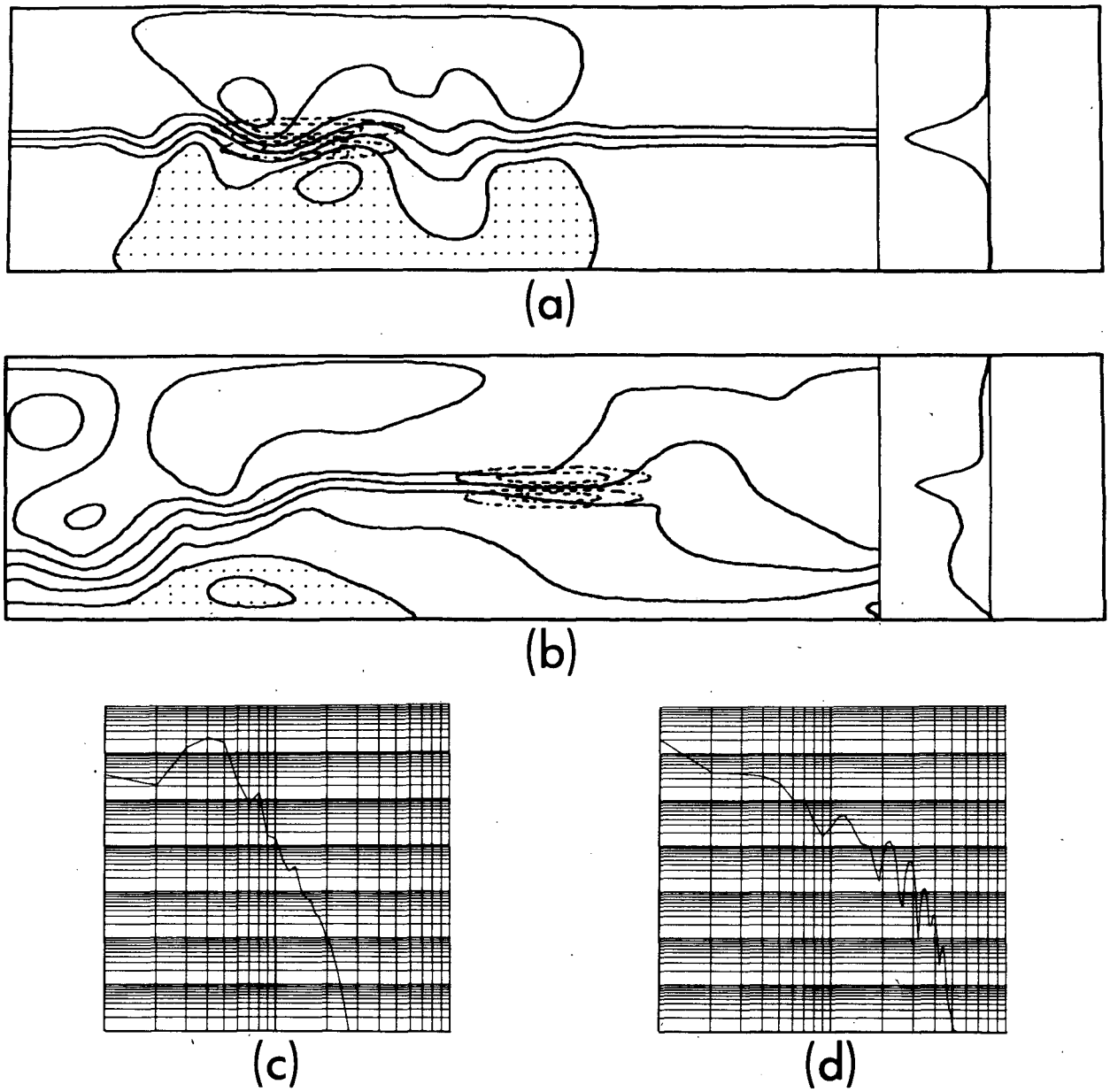

FIG. 3. Case C1: (a) Contours of ψ and profile of $\bar{u}$; dotted lines denote localized forcing. $\Delta\psi=30$ km^2 s^{-1}, $u^*=0.1$ km s^{-1}, $t=11.5$ days; (b) as in Fig. 3 except $u^*=0.05$ km s^{-1}, $t=46.0$ days; (c) normalized zonal spectrum of KE at $\theta=-28°$, $t=11.5$ days; (d) as in Fig. 3c except $t=46.0$ days.

(Table 1). Several simple forms of F are chosen, each one representing a different idea about the processes driving the stratospheric motions (cf. Section 3). In particular, we focus attention on how the flow character depends on the strength and structure of the forcing.

The equilibration of the motion depends on the balance between the drag and the forcing; it occurs when the zonal mean kinetic energy, the eddy kinetic energy and the zonal enstrophy and energy dissipation rates show no secular trend. The values of τ_D are chosen empirically and lie within the characteristic time scales (5–30 days) of the observed variability of UV cloud features (Suomi, 1974; Dollfus, 1975; Traub and Carleton, 1978). Since the forcing and drag time scales are nearly equal in equilibrated flows and since τ_ν exceeds all other time scales, only two flow regimes are possible: those with $\tau_D \ll \tau_0$ or $\tau_D \gg \tau_0$.

a. Strong forcing, $\tau_D \ll \tau_0$

In this regime, the nonlinear kinetic energy cascade process is far less effective than the forcing and drag

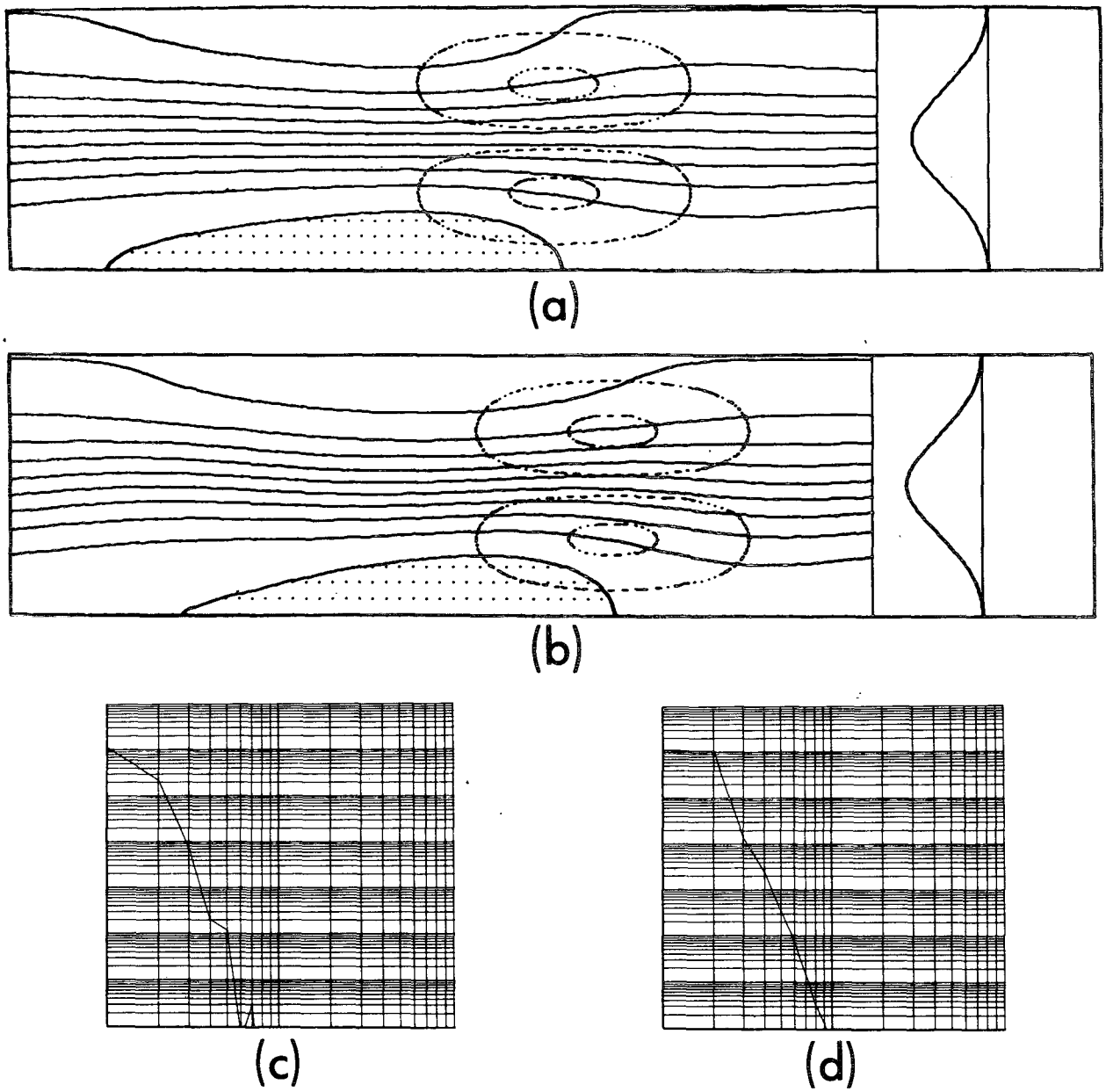

FIG. 4. Case C2: (a) Contours of ψ and profile of $\bar{u}$; dotted lines denote localized forcing. $\Delta\psi = 40$ km^2 s^{-1}, $u^* = 0.1$ km s^{-1}, $t = 46.0$ days; (b) as in Fig. 4a except $t = 54.6$ days; (c) normalized zonal spectrum of KE at $\theta = -28°$, $t = 46.0$ days; (d) as in Fig. 4c except $t = 54.6$ days.

processes in determining the character of the flow. For the two examples selected (Figs. 3 and 4) the flows are generated by a localized forcing (as shown by the dotted lines) that produces an equatorial jet and pair of advection waves and are equilibrated by a drag with a time scale (5 days) comparable to that for motions near Earth's surface. Fig. 3 shows a case where the barotropically unstable, zonal equatorial jet produced by the forcing breaks down into propagating advection waves which interact with the advection waves produced directly by the forcing. However, the nonlinear interactions are so slow compared to the drag that little kinetic energy is transferred to larger scales. Fig. 4 shows a more extreme case where the drag is so strong that the unstable zonal jet does not breakdown and the high-latitude advection waves do not even propagate in the flow, i.e., the forcing produces the exhibited flow directly.

b. Weak forcing, $\tau_D \gg \tau_0$

In this regime, the nonlinear inertial cascade process prevails and motions evolve toward the "relaxed" state

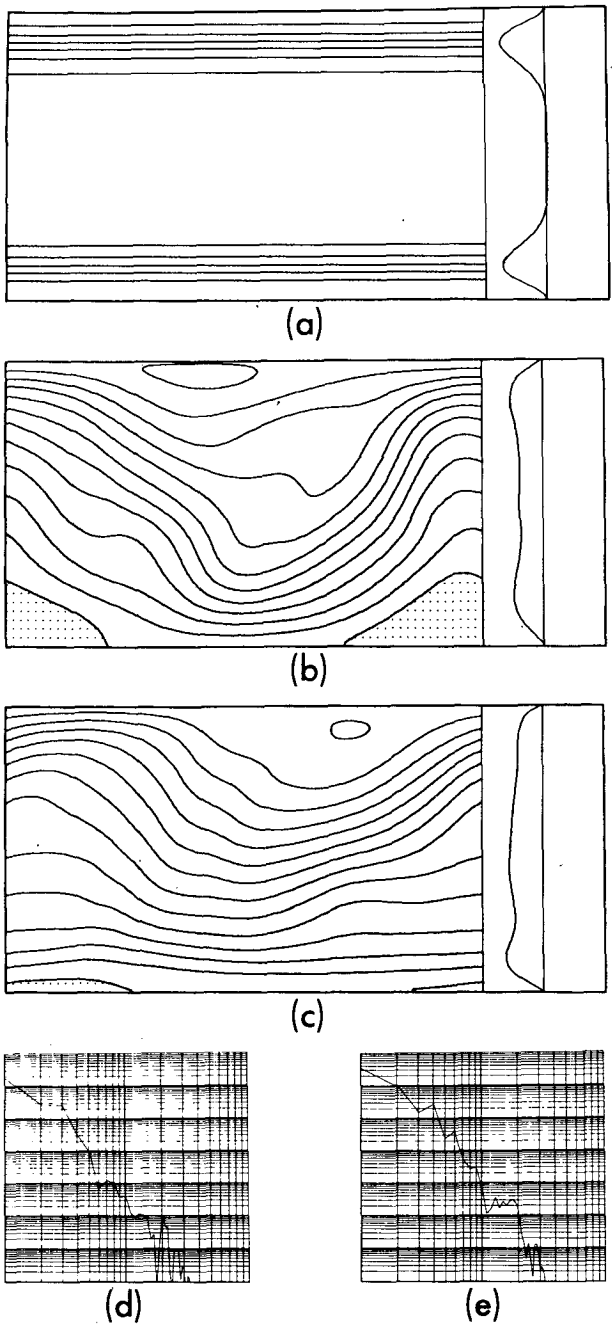

FIG. 5. Case D1: (a) Contours of ψ and profile of $\bar{u}$. $\Delta\psi=30$ km^2 s^{-1}, $u^*=0.1$ km s^{-1}, $t=2.8$ days; (b) as in Fig. 5a except $\Delta\psi=50$ km^2 s^{-1}, $t=11.5$ days; (c) as in Fig. 5b except $t=17.2$ days; (d) normalized zonal spectrum of KE at $\theta=-28°$, $t=11.5$ days; (e) as in Fig. 5d except $t=17.2$ days.

of vorticity conserving flows (cf. Figs. 1 and 2). Figs. 2b and 2e show a situation where the smaller scales of motion are energetic enough to be seen in the streamfunction even though the larger scales of motion dominate the flow. Since Venus' stratosphere lies many scale heights above the surface, large drag forces are not expected, i.e., $\tau_D \gg \tau_0$, so that the dynamics should correspond to that of weakly forced flows.

c. The 4-day wind

The Hadley-cell model (Gierasch, 1975) and gravity-wave model (Fels, 1977) for the 4-day wind imply very different horizontal distributions for the forcing function. In the former model, the poleward advection of a zonal flow by the Hadley cell could produce a strong polar jet if angular momentum were conserved; the shear instability of this jet and the subsequent equatorward transport of angular momentum would

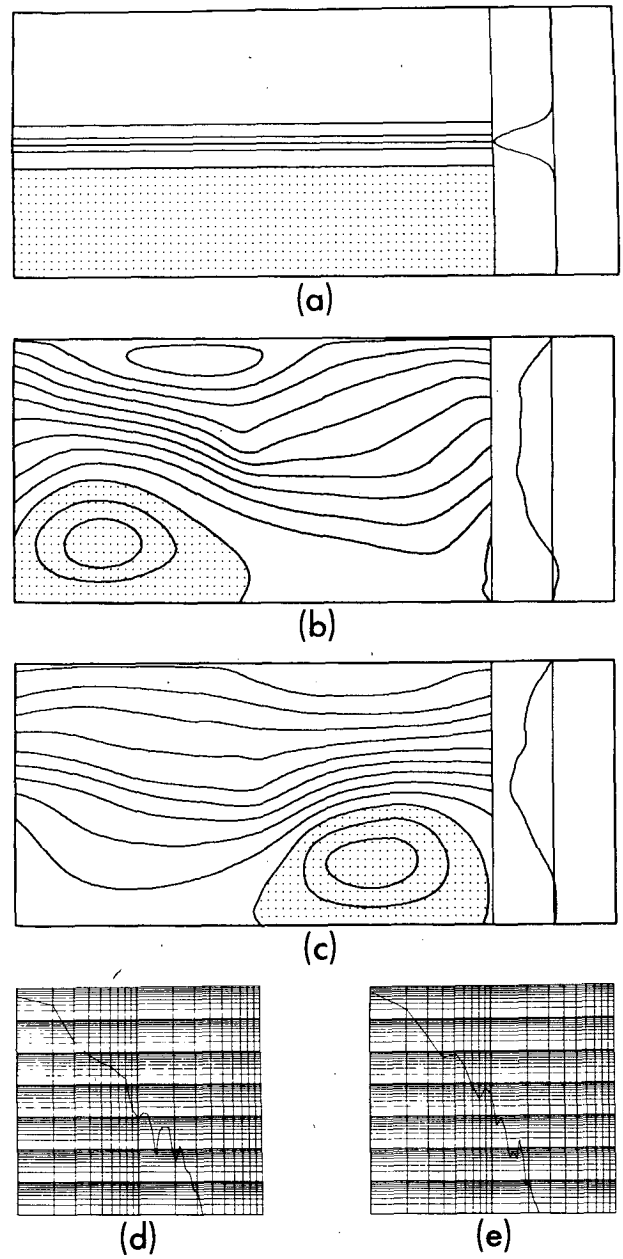

FIG. 6. Case D2: (a) Contours of ψ and profile of $\bar{u}$. $\Delta\psi=30$ km^2 s^{-1}, $u^*=0.12$ km s^{-1}, $t=3.4$ days; (b) as in Fig. 6a except $\Delta\psi=40$ km^2 s^{-1}, $u^*=0.1$ km s^{-1}, $t=20.7$ days; (c) as in Fig. 6b except $t=24.1$ days; (d) normalized zonal spectrum of KE at $\theta=-25.7$, $t=20.7$ days; (e) as in Fig. 6d except $t=24.1$ days.

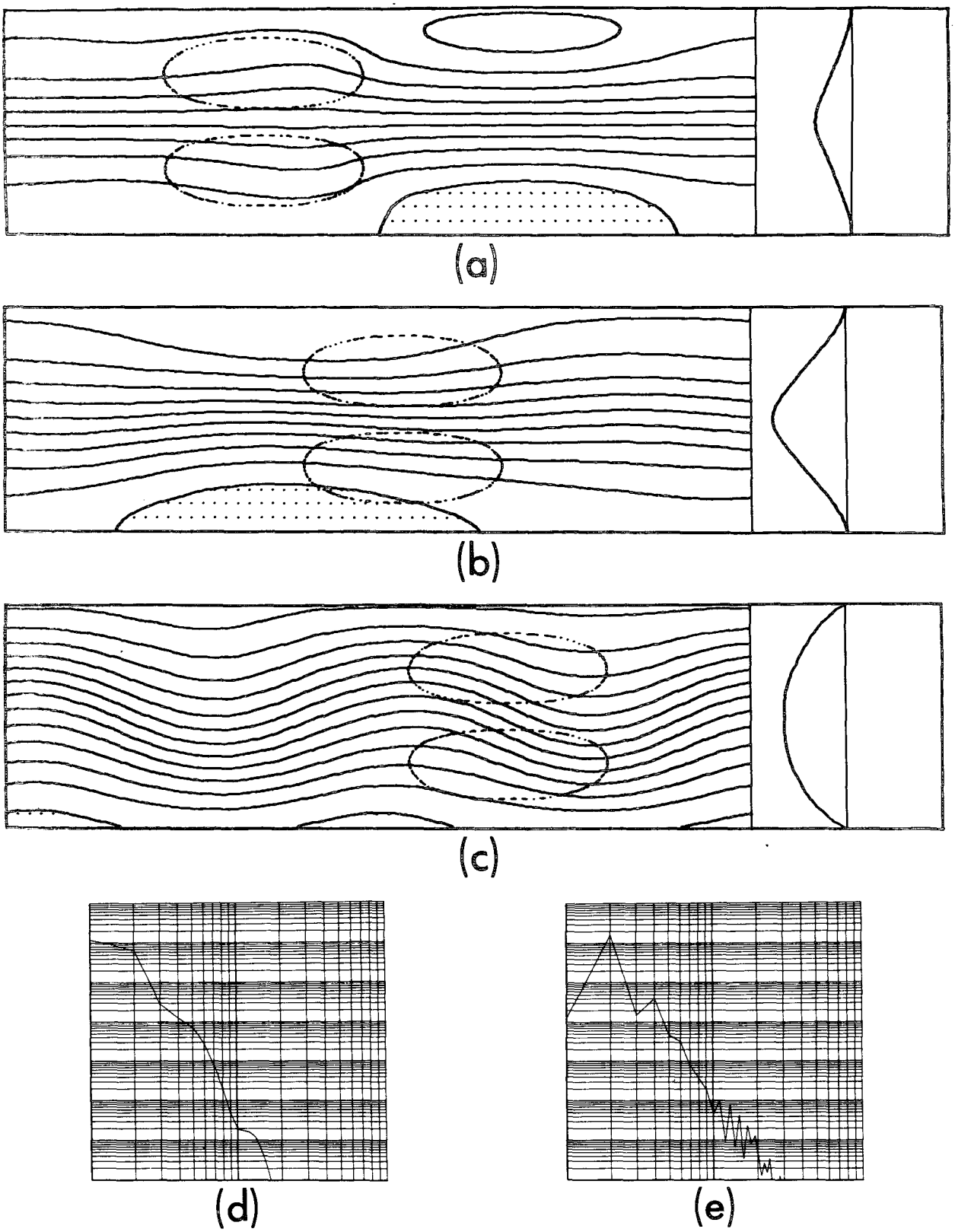

FIG. 7. Case C3: (a) Contours of ψ and profile of $\bar{u}$; dotted lines denote localized forcing. $\Delta\psi = 50$ km^2 s^{-1}, $u^* = 0.2$ km s^{-1}, $t = 11.5$ days; (b) as in Fig. 7a except $\Delta\psi = 100$ km^2 s^{-1}, $t = 34.5$ days; (c) as in Fig. 7b except $t = 51.7$ days; (d) normalized zonal spectrum of KE at $\theta = -28°$, $t = 34.5$ days; (e) as in Fig. 7d except $t = 51.7$ days.

then lead to a net upward transport of planetary momentum by the Hadley cell (Gierasch, 1975). Fig. 5 supports this hypothesis by showing that the nonlinear, two-dimensional exchanges can maintain a nearly constant angular velocity flow, even though the forcing tries to produce polar jets. Thus, these processes, together with a simple Hadley cell, could produce a 4-day wind. Other properties of the flow in Fig. 5, the flat profile of the mean zonal wind with its hint of a midlatitude jet and the midlatitude kinetic energy spectrum with its hint of a "knee" at intermediate wavenumbers, have also been attributed to Venus' UV cloud features (Travis, 1978).

We cannot conclude, however, that we have established the appropriate horizontal distribution for the forcing of the Venus stratosphere, because other distributions of forcing can produce the same type of flow, e.g., one that tends to produce equatorial jets rather than polar jets (Fig. 6). Since the meridional profile of the mean zonal velocity varies in time (cf. Figs. 6b and 6c)—because of the large scale waves—there is really no way to distinguish between this solution and that of Fig. 5 using the limited Mariner 10 photographs. Indeed, we cannot even demonstrate that the forcing must be axisymmetric, as an examination of Fig. 7 shows. Here, a localized forcing, moving with the sun [a possible interpretation of the nature of the forcing in Fels' (1977) model] produces a mean zonal velocity profile and a midlatitude kinetic energy spectrum qualitatively similar to those in Fig. 5. This result is a consequence of the fact that the flow is dominated by inertial effects rather than the forcing and drag effects.

d. The Y feature

The planetary-scale Y-like UV marking which moves around the planet has a highly variable shape and speed (Dollfus, 1975). The production of these UV features and their relation to the clouds and the dynamics is unknown[7] so we can only consider a qualitative comparison between our results and the observed flow. Specifically, the three basic characteristics of the Y—its hemispheric symmetry, longitudinal asymmetry and apparent solid-body motion—coincide with the properties of the stable advection wave in the "relaxed" state of two-dimensional flows. An excellent example of such a wave is shown in Fig. 7; the wave also occurs in Figs. 4–6 but with a smaller amplitude relative to the zonal flow. Comparison of these figures indicates that all horizontal distributions of forcing, whether axisymmetric or localized, produce planetary-scale advection waves in barotropically unstable states. The variability in the flow pattern results from continual excitement of shorter wavelengths by the forcing.

The interpretation of the Y feature as a single global-scale advective wave does not require special conditions for the hemispheric parts to propagate together nor the existence and phase coupling of separate midlatitudinal and equatorial wave modes (cf. Belton *et al.*, 1976b).

6. Conclusions

The motions in the Venus stratosphere are only weakly nondivergent—because of the great efficiency of weak vertical motions in transporting heat in a massive atmosphere and because of the large static stability. Consequently, we have investigated the properties of a one-level atmosphere subject to weak forcing and find that the circulations are dominated by inertial effects—the turbulent cascade of kinetic energy and enstrophy—and that their "relaxed" state resembles that of two dimensional vorticity-conserving flows. On a slowly rotating planet, this "relaxed" state consists of a solid-body rotation plus an advection wave lying symmetrically about the equator and having a zonal wavenumber of unity. Although these large scales predominate, the flow is fully nonlinear in contrast to the quasi-linear Rossby–Haurwitz wave regime occurring on rapidly rotating planets.

The observations of the stratospheric flow on Venus from Mariner 10 are so restricted in spatial and temporal coverage that deducing the nature of the global flow pattern from these data cannot avoid aliasing errors (cf. Belton *et al.*, 1976a) or model fitting (cf. Limaye and Suomi, 1977). The ground based observations have similar problems (Belton *et al.*, 1976a). Given these limitations, the qualitative agreement between interpretations of the Venus' data and the numerical model is satisfactory. However, the data are not of sufficient quality to distinguish between different horizontal distributions of forcing.

Our results have at least two important consequences for further observations of the Venus stratospheric dynamics: 1) because of the global extent of the large-scale waves, and the long, dynamical time scale, systematic global observations over a very long time period are required to obtain the proper statistics of the flow; and 2) when the large-scale circulation is properly described, we still may not be able to deduce the nature of the processes driving these motions because the nonlinear balances of the flow are characteristic of the inertial cascade process rather than the forcing. To isolate the forcing processes will require very accurate, highly resolved observation of the smaller, less energetic scales.

Analogous problems will also arise in further numeri-

[7] For example, pressure variations at constant temperature can control the density and location of condensate clouds. On rapidly rotating planets, the pressure gradients and steamfunction are correlated so that cloud features reveal the streamfunction under these circumstances (cf. Williams, 1979). On Venus, the sulfuric acid clouds are not condensationally active (Rossow, 1978) and the correlation between streamfunction and pressure gradients does not necessarily apply. However, the best correspondence in our results is between the streamfunction and the cloud features.

cal simulation of the Venus stratospheric dynamics. In particular, the proper representation of the inertial cascade process requires relatively high resolution in the model, even though the largest scales dominate the motion. Coarse resolution or severe truncation of the spectrum has two serious effects: 1) it increases the importance of the "eddy diffusion" term for the larger scale motions, making the solution very dependent on a poorly understood parameter (cf. Puri and Bourke, 1974), and 2) it artificially stabilizes some of the planetary scale waves, by removing the smaller wave components required to simulate their nonlinear interactions properly (cf. discussion in Section IV and Baines, 1976). Above all, the solution's lack of dependence on the forcing structure makes it difficult to determine the actual physics by simulation.

In this study, we have not identified the processes producing the four-day wind nor have we examined directly the role played by gravity waves; and perhaps the two questions are related. However, we tentatively conclude that the stratospheric dynamics of Venus is dominated by two-dimensional interactions.

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